

Birthday problem

ways to assign 365
birthdays to k people:
 365^k by multiplication rule.

\Rightarrow "at least 1 match" is
hard b/c you have lots of
ways to make the match.
Instead, use the complement
of "no matches"

$$\Pr[\geq 1 \text{ match}] = 1 - \Pr[\text{no matches}]$$

how to assign 365 birthdays
w/o matches \Leftrightarrow sampling
w/o replacement.

of ways we could have
no matches:

$$365 \times 364 \times \dots \times (365 - k + 1)$$

Use naive def. b/c of
equal likelihood

$$\hookrightarrow \Pr[\text{no match}] = \frac{365 \times \dots \times (365 - k + 1)}{365^k}$$

$$\Pr[\geq 1 \text{ match}] = 1 - \frac{365 \times \dots \times (365 - k + 1)}{365^k}$$

$$k = 23 \Rightarrow \Pr[\geq 1 \text{ match}] > 0.5$$

$$k = 57 \Rightarrow \Pr[\geq 1 \text{ match}] > 0.99$$

Drawing two women senators

$$\Pr[\text{both } W \mid \geq 1 W]$$

both $W \subset \geq 1 W$

$$= \frac{\Pr[\text{both } W \cap \geq 1 W]}{\Pr[\geq 1 W]} = \frac{\Pr[\text{both } W]}{\Pr[\geq 1 W]}$$

17 women $\Rightarrow \binom{17}{2}$ ways
to choose 2 women.

100 senators $\Rightarrow \binom{100}{2}$ ways
to choose 2 senators

$$\Rightarrow \Pr[\text{both } W] = \frac{\binom{17}{2}}{\binom{100}{2}}$$

$$\begin{aligned}\Pr[\geq 1 W] &= 1 - \Pr[\text{no } W] \\ &= 1 - \Pr[\text{both } M]\end{aligned}$$

$\binom{83}{2}$ ways to choose
2 men.

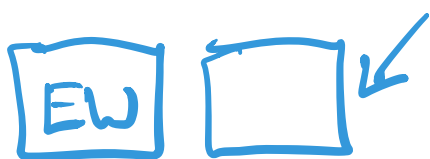
$$\Pr[\geq 1 W] = 1 - \binom{83}{2} / \binom{100}{2}$$

$$\Pr[\text{both } W | \geq 1 W] = \frac{\binom{17}{2} / \binom{100}{2}}{1 - \binom{83}{2} / \binom{100}{2}}$$

$$\approx 0.088$$

$\Pr[\text{both } W | \text{one is E. Warren}]$

Since one is EW we
only need to figure out
the other



16 W left to
choose out of 99 Sens.

$$\Rightarrow \text{Pr}[\text{both } W | \text{one is EW}] = \frac{16}{99} \quad \blacksquare$$