Birthday problem

ways to assign 365 birthdays to k people: 365^k by multiplication rule.

$\Rightarrow$ "at least 1 match" is hard b/c you have lots of ways to make the match. Instead, use the complement of "no matches"

$$\Pr[\geq 1 \text{ match}] = 1 - \Pr[\text{no matches}]$$

how to assign 365 birthdays w/o matches $\Leftrightarrow$ sampling w/o replacement.

# of ways we could have no matches:

$$365 \times 364 \times \cdots \times (365-k+1)$$

Use naive def. b/c of equal likelihood

$$\hookrightarrow \Pr[\text{no match}] = \frac{365 \times \cdots \times (365-k+1)}{365^k}$$

$$\Pr[\geq 1 \text{ match}] = 1 - \frac{365 \times \cdots \times (365-k+1)}{365^k}$$

$$k = 23 \Rightarrow \Pr[\geq 1 \text{ match}] > 0.5$$

$$k = 57 \Rightarrow \Pr[\geq 1 \text{ match}] > 0.99$$

Drawing two women senators

$\Pr[\text{both W} \mid \geq 1 \text{ W}]$

$\boxed{\text{both W} \subset \geq 1 \text{ W}}$

$$= \frac{\Pr[\text{both W} \cap \geq 1 \text{ W}]}{\Pr[\geq 1 \text{ W}]} = \frac{\Pr[\text{both W}]}{\Pr[\geq 1 \text{ W}]}$$

17 women $\Rightarrow \binom{17}{2}$ ways to choose 2 women.

100 senators $\Rightarrow \binom{100}{2}$ ways to choose 2 senators

$\Rightarrow \Pr[\text{both W}] = \binom{17}{2} \Big/ \binom{100}{2}$

$$\Pr[\geq 1 \text{ W}] = 1 - \Pr[\text{no W}]$$
$$= 1 - \Pr[\text{both M}]$$

$\binom{83}{2}$ ways to choose 2 men.

$$\Pr[\geq 1 W] = 1 - \binom{83}{2} \Big/ \binom{100}{2}$$

$$\Pr[\text{both } W \mid \geq 1 W] = \frac{\binom{17}{2} \big/ \binom{100}{2}}{1 - \binom{83}{2} \big/ \binom{100}{2}}$$

$$\approx 0.088$$

$\Pr[\text{both } W \mid \text{one is E. Warren}]$

since one is EW we only need to figure out the other

$\boxed{EW}$ $\boxed{}$ 16 W left to choose out of 99 Sens.

$$\Rightarrow \Pr[\text{both W} \mid \text{one is EW}] = \frac{16}{99}$$