

$$X \sim \text{Bin}(n, p) \quad n=6$$

$$\underline{100110} \quad X=3$$

$$\text{Pr}[\underline{100110}] = \text{Pr}[1] \text{Pr}[0] \dots \text{Pr}[0]$$

independent

$$= p(1-p)(1-p)p(1-p)$$

$$= p^3(1-p)^3$$

$$\underline{100000} \Rightarrow p^1(1-p)^5$$

more generally,

$$\underline{X=k}$$

$$p^k (1-p)^{n-k}$$

$q=1-p$

$$\underline{111000} = p^3(1-p)^3$$

how many ways to
get k successes
in n trials

$$\frac{1}{1} \frac{1}{2} \frac{1}{3} \frac{1}{4} \frac{1}{5} \frac{1}{6}$$

$$\left(\frac{n(n-1)\dots(n-k+1)}{k!} \right) \binom{n}{k}$$

$$= \frac{n!}{k!(n-k)!}$$

$$\underline{P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}}$$

Binomial thm

$$(x+y)^n = \sum_{k=0}^n \binom{n}{k} x^k y^{n-k}$$

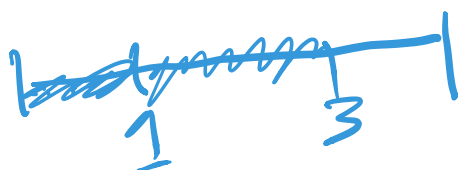
$$(a+b)^2 = a^2 + 2ab + b^2$$

$$\sum_{k=0}^n \Pr[X=k]$$

$$= \sum_{k=0}^n \binom{n}{k} p^k (1-p)^{n-k}$$

$$= (p + 1-p)^n = 1^n = 1$$

CDF


$$Pr[1 < X \leq 3]$$

$$Pr[X \leq 3] = Pr[\{X \leq 1\} \cup \{1 < X \leq 3\}]$$
$$= Pr[X \leq 1] + Pr[1 < X \leq 3]$$

$$Pr[1 < X \leq 3]$$
$$= Pr[X \leq 3] - Pr[X \leq 1]$$
$$\underline{F(3)} - \underline{F(1)}$$

$$Pr[X > 3] = 1 - Pr[\{X > 3\}^c]$$

~~1 - Pr[X > 3]~~

$$\begin{aligned} & 3 \\ & = 1 - \Pr[X \leq 3] \\ & = 1 - F(3) \end{aligned}$$

$$X = X_1 + \dots + X_n$$

$$Y = Y_1 + \dots + Y_m$$

$$\underbrace{X+Y}_{\text{sum of } n+m \text{ iid.}} = X_1 + \dots + X_n + Y_1 + \dots + Y_m$$

↳ sum of $n+m$ iid.

Bern(p)

$\sim \text{Bin}(n+m, p)$