$X \sim \mathrm{Bin}(n, p)$ $n = 6$

$\underline{100110}$ $X = 3$

$\Pr[100110] = \Pr[1]\Pr[0]\cdots\Pr[0]$

independent

$= p(1-p)(1-p)pp(1-p)$

$= p^3(1-p)^3$

$\underline{100000} \Rightarrow p^1(1-p)^5$

$q = 1-p$

more generally,

$\underline{X = k}$ $p^k(1-p)^{n-k}$

$\underline{111}\,\underline{000} = p^3(1-p)^3$

how many ways to get k successes in n trials

$$\frac{1}{1} \quad \frac{1}{2} \quad \frac{1}{3} \quad \frac{}{4} \quad \frac{}{5} \quad \frac{}{6}$$

$$\left(\frac{n(n-1)\cdots(n-k+1)}{k!} \right) = \binom{n}{k}$$

$$= \frac{n!}{k!\,(n-k)!}$$

$$\Pr[X=k] = \binom{n}{k} \underbrace{p^k (1-p)^{n-k}}$$

Binomial thm

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$\sum_{k=0}^{n} \Pr[X=k]$$

$$= \sum_{k=0}^{n} \binom{n}{k} p^k (1-p)^{n-k}$$

$$= (p+1-p)^n = 1^n = 1$$

CDF

$$\Pr[1 < X \le 3]$$

$$\Pr[X \le 3] = \Pr[\{X \le 1\} \cup \{1 < X \le 3\}]$$

$$= \Pr[X \le 1] + \underbrace{\Pr[1 < X \le 3]}$$

$$\Pr[1 < X \le 3]$$

$$= \Pr[X \le 3] - \Pr[X \le 1]$$

$$\underline{F(3)} - \underline{F(1)}$$

$$\Pr[X > 3] = 1 - \Pr[\{X > 3\}^c]$$

$$= 1 - \Pr[X \leq 3]$$

$$= 1 - F(3)$$

$$X = X_1 + \cdots + X_n$$

$$Y = Y_1 + \cdots + Y_m$$

$$\underbrace{X + Y} = X_1 + \cdots + X_n + Y_1 + \cdots + Y_m$$

$\hookrightarrow$ sum of $n+m$ iid. Bern(p)

$$\sim \text{Bin}(n+m, p)$$