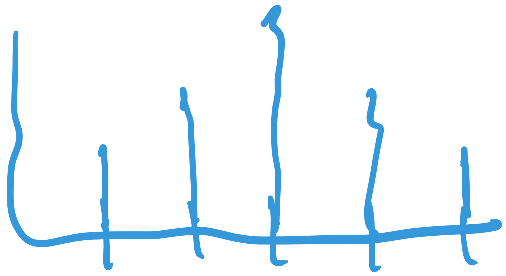
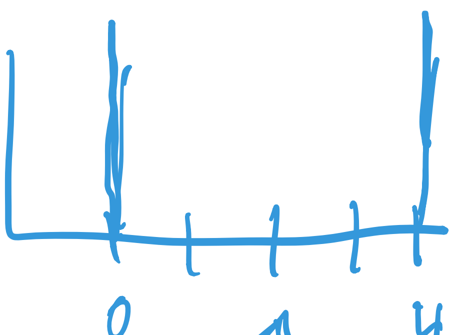


$E[X]$



$E[Y]$

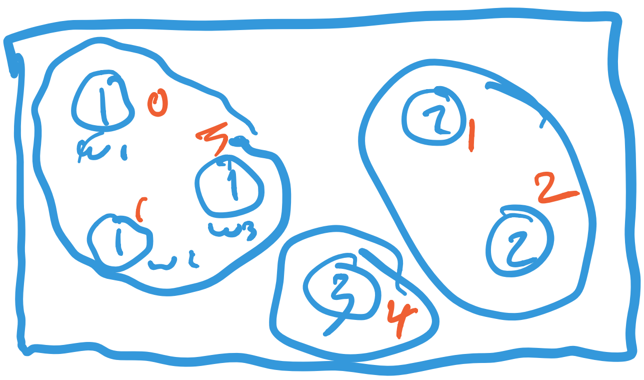
$$\underbrace{E[X+Y]} = \underbrace{E[X] + E[Y]}$$

$$E[X+Y] = \sum_t t \, \underline{\Pr[X+Y=y]}$$

$$\overset{LTP}{=} \sum_t \sum_x t \Pr[X+Y=t \mid X=x] \Pr[X=x]$$

$$= \sum_t \sum_x t \Pr[Y=t-x \mid X=x] \Pr[X=x]$$

$$\sum_x x \Pr[X=x] + \sum_y y \Pr[Y=y]$$



$$\Pr[\{X=1\}] = \Pr[\omega_1] + \Pr[\omega_2] + \Pr[\omega_3]$$

$$E[X] = \sum_{\omega \in \Omega} X(\omega) \Pr[\omega]$$

$$E[Y] = \sum_{\omega \in \Omega} Y(\omega) \Pr[\omega]$$

$$E[X+Y] = \sum_{\omega \in \Omega} (X+Y)(\omega) \Pr[\omega]$$

$$= \sum_{\omega \in \Omega} \left(X(\omega) + Y(\omega)\right) \Pr[\omega]$$

$$= \sum_{\omega} X(\omega) \Pr[\omega] + \sum_{\omega} Y(\omega) \Pr[\omega]$$

$$= E[X] + E[Y]$$

$$E[X] \simeq \frac{1}{n} \sum_i x_i$$

$$E[X+Y] = \frac{1}{n} \sum_i (x_i + y_i)$$

$$= \frac{1}{n}\sum_i x_i + \frac{1}{n}\sum_i y_i$$

$$= E[X] + E[Y]$$

$$Z = X - Y$$

$$P([Z \geq 0] = 1$$

$$E[Z] \geq 0$$

$$E[X - Y] \geq 0$$

$$E[X] - E[Y] \geq 0$$

$$E[X] \geq E[Y]$$