$$E[(X - E[X])^2]$$

$$= E[X^2 - 2XE[X] + E[X]^2]$$

$$= E[X^2] - 2E[X]E[X] + E[X^2]$$

$$= E[X^2] - (E[X])^2$$

Pois has a valid p.m.f.

$$\sum_{k=0}^{\infty} \frac{e^{-\lambda}\lambda^k}{k!} = e^{-\lambda}\sum_{k=0}^{\infty}\frac{\lambda^k}{k!}$$

$$= e^{-\lambda}e^{\lambda} = 1$$

$E[X]$ if $X \sim Pois(\lambda)$

$$E[X] = \sum_{k=0}^{\infty} k \frac{e^{-\lambda}\lambda^k}{k!}$$

$$= e^{-\lambda} \sum_{k=1}^{\infty} k \frac{\lambda^k}{k!}$$

$$\underline{\cancel{k}(k-1)(k-2)\cdots 1}$$

$$= e^{-\lambda} \underline{\sum_{k=1}^{\infty} \frac{\lambda^k}{(k-1)!}} = e^{-\lambda} \sum_{k=1}^{\infty} \lambda \frac{\lambda^{k-1}}{(k-1)!}$$

$$= \lambda e^{-\lambda} \underline{\sum_{k=1}^{\infty} \frac{\lambda^{k-1}}{(k-1)!}} = \lambda \cancel{e^{-\lambda} e^{\lambda}}$$

$$= \lambda$$

$$V[X] = \underbrace{E[X^2]} - (\underbrace{E[X]}_{\lambda})^2$$

$$\lambda^2$$

$$E[X^2] = \sum_{k=0}^{\infty} k^2 \frac{e^{-\lambda}\lambda^k}{k!}$$

$$= e^{-\lambda} \sum_{k=1}^{\infty} k \frac{\lambda^k}{(k-1)!}$$

$$= e^{-\lambda} \sum_{k=1}^{\infty} (k-1+1) \frac{\lambda^k}{(k-1)!}$$

$$= \lambda \left[e^{-\lambda} \sum_{k=1}^{\infty} (k-1) \frac{\lambda^{k-1}}{(k-1)!} \right]$$

$$+ \lambda e^{-\lambda} \underline{\sum_{k=1}^{\infty} \frac{\lambda^{k-1}}{(k-1)!}}$$

$$= \lambda E[X] + \lambda e^{-\lambda} e^{\lambda}$$

$$= \lambda^2 + \lambda$$

$$V[X] = \lambda^2 + \lambda - \lambda^2 = \lambda$$