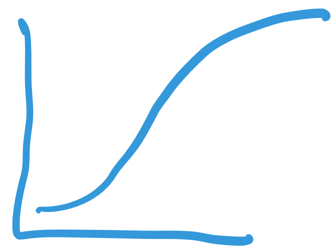
Logistic r.v. X has a CDF that is the logistic function.

$$F(x) = \frac{e^x}{1+e^x}$$



$$f(x) = F'(x)$$

ratio => quotient rule

$$F(x) = \frac{h(x)}{g(x)}$$

$$F'(x) = \frac{h'(x)g(x) - h(x)g'(x)}{g(x)^2}$$

$$f(x) = F'(x) = \frac{e^x(1+e^x) - e^x e^x}{(1+e^x)^2}$$

$$f(x) = \frac{e^x}{(1+e^x)^2}$$

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Uniform $(a,b) = U$

$$1 = \Pr[U \in (a,b)]$$

$$= \int_a^b c\,dx = c\int_a^b 1\,dx$$

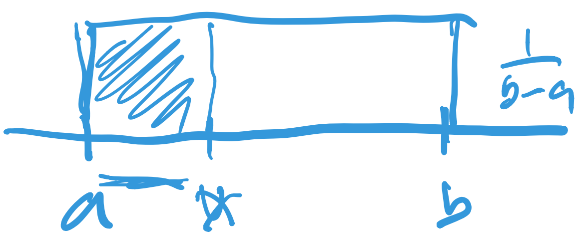
$$= cx\Big|_a^b = \underline{c(b-a) = 1}$$

$$\underline{c = \frac{1}{b-a}}$$

$U \sim \text{Unif}(a,b)$

$$\underline{\Pr[U \le \omega \mid U \in (c,d)]}$$

c.d.f. $\Rightarrow$ $\tilde{U} \sim \text{Unif}(c,d)$



$$\Pr[U \le x] = \frac{x-a}{b-a} = \overset{\ell}{x-a} \times \overset{h}{\left(\frac{1}{b-a}\right)}$$

$$U \sim \text{Unif}(a,b)$$

$$E[U] = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \int_a^b x \frac{1}{b-a}\,dx \qquad \frac{\partial x^2}{\partial x} = 2x$$

$$= \frac{1}{b-a}\frac{1}{2}x^2\Big|_a^b = \frac{1}{2(b-a)}(b^2 - a^2)$$

$$= \frac{1}{2\cancel{(b-a)}}\cancel{(b-a)}(b+a)$$

$$= \frac{b+a}{2} = \underline{E[U]}$$

$R \sim \text{Unif}(0,1)$
$A = \pi R^2$

$E[A] = E[\pi R^2]$

$= \int_0^1 \pi r^2 \underbrace{1}\, dr = \pi \int_0^1 r^2\, dr$

$= \frac{\pi}{3} r^3 \Big|_0^1 = \frac{\pi}{3} - 0 = \frac{\pi}{3}$

$\underline{V[A] = \underline{E[A^2]} - \underline{E[A]}^2}$

$E[A^2] = E[\pi^2 R^4]$

$$= \int_0^1 \pi^2 r^4 dr = \pi^2 \int_0^1 r^4 dr$$

$$= \frac{\pi^2}{5} r^5 \Big|_0^1 = \frac{\pi^2}{5}$$

$$V[A] = \frac{\pi^2}{5} - \frac{\pi^2}{9}$$

$$= \frac{4\pi^2}{45}$$