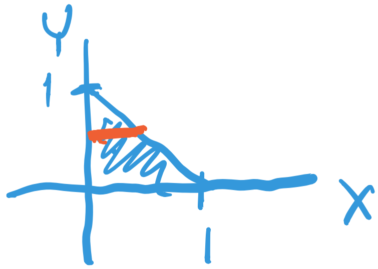
Ex Unif distribution

triangle $(0,0) - (1,0) - (0,1)$



$x, y \geq 0$

$x + y \leq 1$

PDF c over

$$1 = \int_0^1 \underbrace{\int_0^{1-y} c \, dx}_{} \, dy$$

$$= \int_0^1 (cx)\Big|_0^{1-y} dy$$

$$= \int_0^1 c(1-y) \, dy$$

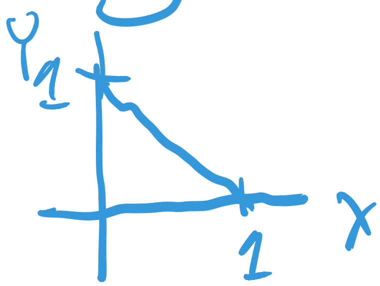
$$= c\left(y - \tfrac{1}{2}y^2\right)\Big|_0^1$$

$$= \frac{c}{2} \Rightarrow c = 2$$

PDF

$$\boxed{f_{XY}(x,y) = 2 \text{ when } x, y \geq 0 \quad x+y \leq 1}$$

marginal $\underline{f_X(x)}$



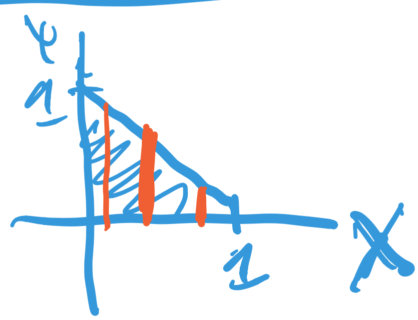
$$f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dy$$

$$= \int_0^{1-x} 2\,dy$$

$$= 2y \Big|_0^{1-x}$$

$$f_X(x) = 2 - 2x$$

By symmetry, $f_Y(y) = 2 - 2y$



$$f_{Y|X}(y|x) = \frac{f_{XY}(x,y)}{f_X(x)}$$

$$= \frac{2}{2(1-x)} = \frac{1}{1-x}$$

for $y \in (0, 1-x)$

$$f_{Y|X}(y|x) = \frac{1}{1-0.5} = 2$$

for $y \in (0, \tfrac{1}{2})$

$$Y \mid X = x \sim \text{Unif}(0, 1-x)$$

$$E[Y] = \sum_x \sum_y y \Pr[X=x, Y=y]$$

$$= \sum_y \sum_x y \Pr[X=x, Y=y]$$

$$= \sum_y y \underbrace{\sum_x \Pr[X=x, Y=y]}$$

$$= \underbrace{\sum_y y \Pr[Y=y]}$$

$$E[XY] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy\, f_{XY}(x,y)\,dx\,dy$$

$$= \int_{-\infty}^{\infty} \underbrace{\int_{-\infty}^{\infty} xy\, f_X(x) f_Y(y)\,dx}\, dy$$

$$= \int_{-\infty}^{\infty} y f_Y(y) \underbrace{\int_{-\infty}^{\infty} x f_X(x)\,dx}_{E[X]}\, dy$$

$$= E[X] \underbrace{\int_{-\infty}^{\infty} y f_Y(y)\,dy}$$

$$= E[X]E[Y]$$

$$E[(X+Y)Z] - \underbrace{E[(X+Y)]}E[Z]$$

$$E[XZ] + E[YZ] - E[X]E[Z]$$

$$t - E[Y]\,E[Z]$$

$$V[X-Y] = V[X+(-Y)]$$

$$= V[X] + V[-Y]$$

$$= V[X] + (-1)^2 V[Y]$$

$$= V[X] + V[Y]$$

$$X = \begin{cases} 1 & \text{w.p. } 1/3 \\ 0 & \text{w.p. } 1/3 \\ -1 & \text{w.p. } 1/3 \end{cases} \qquad Y = X^2$$

$$E[X] = 0$$

$$Cov[X,Y] = E[XX^2] - E[X]\,E[X^2]$$

$$= E[X^3] - \underbrace{E[X]}_{0} E[X^2]$$

$$X^3 = X$$

$$E[X^3] = 0$$

$$\mathrm{Cov}(X, Y) = 0$$

$$X, \quad Y = X^2$$