$Z \sim N(0,1)$

$E[Z] = 0$

$$E[Z] = \int_{-\infty}^{\infty} z \underline{\varphi(z)} \, dz$$

$$= \int_{-\infty}^{0} z \varphi(z) \, dz + \int_{0}^{\infty} z \varphi(z) \, dz$$

$$= \int_{0}^{\infty} (-z) \varphi(-z) \, dz + \int_{0}^{\infty} z \varphi(z) \, dz$$

$$= -\int_{0}^{\infty} z \varphi(z) \, dz$$

$$+ \int_{0}^{\infty} z \varphi(z) \, dz = 0$$

CDF of $X \sim N(\mu, \sigma^2)$

$$F(x) = \Pr[X \le x]$$

$$= \Pr[X - \mu \le x - \mu]$$

$$= \Pr\Big[\underbrace{\frac{X-\mu}{\sigma}}_{N(0,1)} \le \frac{x-\mu}{\sigma}\Big]$$

$$= \Pr\Big[Z \le \frac{x-\mu}{\sigma}\Big]$$

$$= \Phi\Big(\frac{x-\mu}{\sigma}\Big)$$

$$f(x) = F'(x) = \frac{\partial}{\partial x}\Phi\Big(\frac{x-\mu}{\sigma}\Big)$$

$$= \underline{\varphi\Big(\frac{x-\mu}{\sigma}\Big)}\frac{1}{\sigma}$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

$$X \sim N(1, 4)$$

$$\Pr[|X| \leq 3]$$

$$= \Pr[-3 \leq X \leq 3]$$

$$= \Pr[-3-1 \leq X-1 \leq 3-1]$$

$$= \Pr\left[\frac{-3-1}{2} \leq \frac{X-1}{2} \leq \frac{3-1}{2}\right]$$

$$= \Pr[-2 \leq Z \leq 1]$$

$$\searrow \quad \Phi(1) - \Phi(-2)$$

$$T = \frac{\bar{X} - \mu}{\sqrt{S^2/n}} \qquad X_i \overset{iid}{\sim} N(\mu, \sigma^2)$$

$$V = \frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$$

$$= \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \times \frac{1}{\sqrt{\frac{S^2}{\sigma^2}}} \qquad \sqrt{\frac{S^2}{\sigma^2}} \sim \sqrt{\frac{V}{n-1}}$$

$$\sim \underbrace{\frac{\bar{X} - \mu}{\sigma/\sqrt{n}}}_{N(0,1)} \times \frac{1}{\sqrt{\underbrace{V}_{\chi^2_{n-1}}/(n-1)}} \qquad V[\bar{X}] = \frac{\sigma^2}{n}$$

$$\sim t_{n-1}$$

$$V\left[(X_1\ X_2)'\right]$$

$$= E\left[\underset{2\times 1}{\begin{pmatrix} X_1 - E[X_1] \\ X_2 - E[X_2] \end{pmatrix}} \underset{1\times 2}{\left(X_1 - E[X_1],\ X_2 - E[X_2]\right)} \right]$$

$$= E\begin{bmatrix} (X_1 - E[X_1])^2 & (X_1 - E[X_1])(X_2 - E[X_2]) \\ (X_2 - E[X_2])(X_1 - E[X_1]) & (X_2 - E[X_2])^2 \end{bmatrix}$$

$Z: Z_1, \dots, Z_K \Rightarrow$ p.d.f of Z

$$f(z_1, z_2, \dots, z_K)$$

$$= f(z_1) f(z_2) \dots f(z_K)$$

$$= \prod_{i=1}^{K} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z_i^2}{2}\right)$$

$$= \frac{1}{(2\pi)^{K/2}} \exp\left(-\frac{1}{2} \sum_{i=1}^{K} z_i^2\right)$$

$$\sum_{i=1}^{K} z_i^2 = \underline{z'z}$$

$$= \frac{1}{(2\pi)^{K/2}} \exp\left(-\frac{z'z}{2}\right)$$