Weighted average

w_i $\qquad \bar{X}_w = \dfrac{\sum_{i=1}^n w_i X_i}{\sum_{i=1}^n w_i}$

$X_1, \dots, X_n$ i.i.d. $\quad E[X_i] = \mu$

$$E[\bar{X}_w] = E\left[\frac{\sum_{i=1}^n w_i X_i}{\sum_{i=1}^n w_i}\right]$$

$$= \frac{1}{\sum_{i=1}^n w_i} \sum_{i=1}^n w_i E[X_i]$$

$$= \frac{1}{\sum_{i=1}^n w_i} \sum_{i=1}^n w_i \underline{\mu}$$

$$= \frac{\mu \cancel{\left(\sum_{i=1}^n w_i\right)}}{\cancel{\left(\sum_{i=1}^n w_i\right)}} = \underline{\underline{\mu}}$$

estimating σ^2

$X_1, \dots, X_n$ iid

$E[X_i] = \mu$

$V[X_i] = \sigma^2$

$$\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} (X_i - \bar{X}_n)^2$$

μ

$$\bar{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} (X_i - \mu)^2$$

$$E[\bar{\sigma}^2] = \frac{1}{n} \sum_{i=1}^{n} E[(X_i - \mu)^2]$$

$$= \frac{1}{n} \sum_{i=1}^{n} \sigma^2$$

$$= \sigma^2$$

$$\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} (X_i - \bar{X})^2$$

$$\frac{1}{n} \sum_{i=1}^{n} (X_i - c)^2$$

$$= \frac{1}{n} \sum_{i=1}^{n} (\underbrace{X_i - \bar{X}}_{a} + \underbrace{\bar{X} - c}_{b})^2$$

$$= \frac{1}{n} \sum_i (X_i - \bar{X})^2 + \frac{2}{n} \sum_{i=1}^{n} (X_i - \bar{X})(\bar{X} - c)$$

$$+ \frac{1}{n}\sum_{i=1}^{n}(\bar{X}_n - c)^2$$

$$= \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 + (\bar{x} - c)^2$$

$$\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2 - (\bar{x} - \mu)^2$$

$$= \bar{\sigma}^2 - (\bar{x} - \mu)^2$$

$$E[\hat{\sigma}^2] = E[\bar{\sigma}^2] - E[(\bar{x} - \mu)^2]$$

$$= \sigma^2 - V[\bar{X}]$$

$$= \sigma^2 - \frac{\sigma^2}{n}$$

$$= \left(1 - \frac{1}{n}\right)\sigma^2 = \left(\frac{n-1}{n}\right)\sigma^2$$

$$E\left[\left(\frac{n}{n-1}\right)\hat{\sigma}^2\right] = \sigma^2$$

$$S^2 = \left(\frac{n}{n-1}\right)\hat{\sigma}^2$$

$$= \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$$

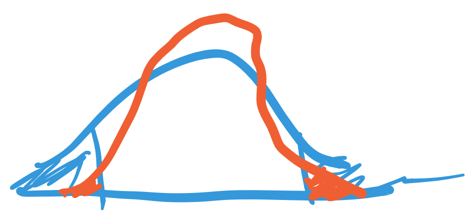
<u>Chebyshev</u>

$$Z = X - E[X]$$

$$E[Z] = 0$$

$$E[Z^2] = V[Z] = V[X]$$

$$Pr\left[\,|Z| \ge \delta\,\right]$$



$-\delta \qquad \delta$

$$= \int_{|x|\ge\delta} 1\, f_Z(x)\,dx$$

$$|x| \ge \delta \Rightarrow \underline{\frac{x^2}{\delta^2}} \ge \underline{1}$$

$$\leq \int_{|x| \geq \delta} \frac{x^2}{\delta^2} f_Z(x)\, dx$$

$$\leq \frac{1}{\delta^2} \int_{-\infty}^{\infty} x^2 f_Z(x)\, dx$$

$$= \frac{1}{\delta^2} E[Z^2]$$

$$= \frac{V[X]}{\delta^2}$$

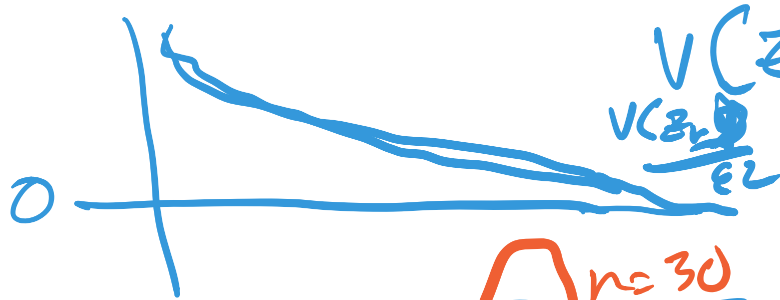
$$\underline{0 \leq Pr[\,|Z_n - \mu| \geq \varepsilon\,]} \leq \frac{V[Z_n - \mu]}{\varepsilon^2}$$

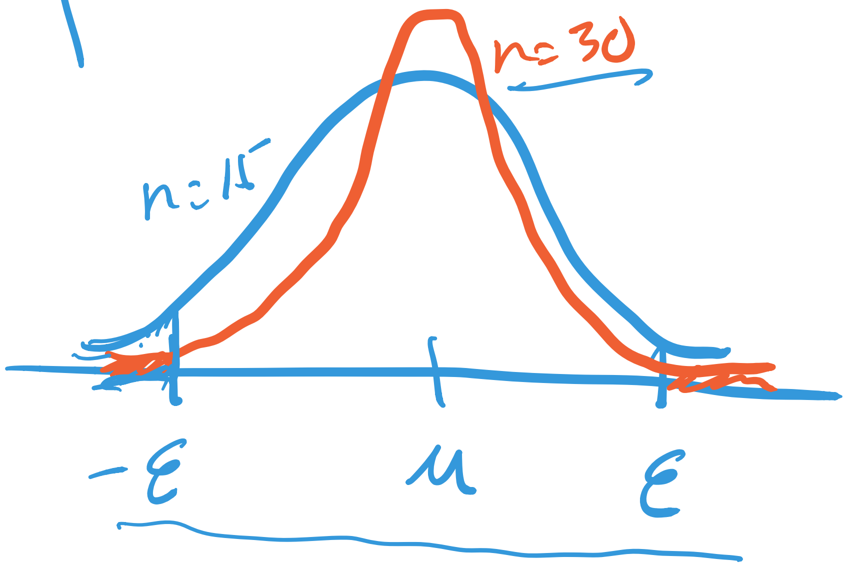
$$Z_n \xrightarrow{P} \mu$$

$$Z_n - \mu \xrightarrow{P} 0$$

$$= \frac{V[Z_n]}{\varepsilon^2}.$$

$V[Z_n] \to 0$





$P\left(\left[Z_n \to \mu \right]\right) = 1$