$$\sqrt{n}(\bar{X}_n - \mu) \xrightarrow{d} N(0, \sigma^2)$$

$$X_i \overset{iid}{\sim} \quad \left. \begin{array}{l} E[X_i] = \mu \\ V[X_i] = \sigma^2 \end{array} \right\}$$

$$\sqrt{n}(\bar{X}_n^2 - \mu^2)$$

$$h(\mu) = \mu^2$$

$$\underline{h'(\mu)} = 2\mu$$

$$\sqrt{n}(\underline{\bar{X}_n^2} - \mu^2) \xrightarrow{d} N(0, \underline{(2\mu)^2\sigma^2})$$

$$V[\bar{X}_n^2] = 4\mu^2\sigma^2$$

$$\theta = E[g(X_i)]$$

$X_1, \dots, X_n \overset{iid}{\sim}$ $\quad E[g(X_i)^2] < \infty$

$$\hat{\theta}_n = \frac{1}{n} \sum_{i=1}^{n} g(X_i)$$

by CLT,

$$\underline{\sqrt{n}(\hat{\theta}_n - \theta) \xrightarrow{d} N(0, V_\theta)}$$

$$V_\theta = V[g(X_i)]$$

$$= E[(g(X_i) - \theta)^2]$$

$$\hat{V}_\theta = \frac{1}{n} \sum_{i=1}^{n} (g(X_i) - \hat{\theta}_n)^2$$

$$= \Big(\underbrace{\frac{1}{n} \sum_i g(X_i)^2}_{\downarrow p}\Big) - \hat{\theta}_n^2$$

$$\xrightarrow{p} E[g(X_i)^2] - \theta^2$$

WLLN

$$= E[(g(X_i) - \theta)^2]$$

$$= V_\theta$$

$$\hat{V}_\theta \xrightarrow{p} V_\theta$$

$$\frac{\sqrt{n}(\hat{\theta}_n - \theta)}{\sqrt{\hat{V}_\theta}} \xrightarrow{d} N(0,1)$$

$$= \underbrace{\frac{\sqrt{V_\theta}}{\sqrt{\hat{V}_\theta}}}_{\downarrow p \atop 1} \times \underbrace{\frac{\sqrt{N}(\hat{\theta}_n - \theta)}{\sqrt{V_\theta}}}_{\downarrow d \atop N(0,1)}$$

$$\frac{\sqrt{V_\theta}}{\sqrt{\hat{V}_\theta}} \xrightarrow{p} \frac{\cancel{\sqrt{V_\theta}}}{\cancel{\sqrt{V_\theta}}} = 1$$

$$h(\theta_1, \theta_2, \theta_3) = \begin{pmatrix} \theta_2/\theta_1 \\ \theta_3/\theta_1 \end{pmatrix}$$

$$H(\theta) = \begin{pmatrix} -\theta_2/\theta_1^2 & 1/\theta_1 & 0 \\ -\theta_3/\theta_1^2 & 0 & 1/\theta_1 \end{pmatrix}$$

$$\hat{\theta}_n = (\bar{X}_1, \bar{X}_2, \bar{X}_3)$$

$$\sqrt{n}\,(\hat{\theta}_n - \theta) \xrightarrow{d} N(0, \Sigma)$$

$$\sqrt{n}\left(\begin{pmatrix}\bar{X}_2/\bar{X}_1\\ \bar{X}_3/\bar{X}_1\end{pmatrix}-\begin{pmatrix}\theta_2/\theta_1\\ \theta_3/\theta_1\end{pmatrix}\right)$$

$$\xrightarrow{d} N(0, H\Sigma H^{-1})$$

$$\Sigma = \begin{pmatrix}\sigma_1^2 & \sigma_{21} & \sigma_{31}\\ \sigma_{12} & \sigma_2^2 & \sigma_{23}\\ \sigma_{13} & \sigma_{23} & \sigma_3^2\end{pmatrix}$$

$$E[X_i] = \mu$$

$$V[X_i] = \sigma^2$$

$$E[\bar{X}_n] = \mu$$

$$V[\bar{X}_n] = \sigma^2/n$$

$$Y = \bar{X}_n - \mu \Rightarrow E[Y] = 0$$

$$V[Y] = \frac{\sigma^2}{n}$$

$$Z = \sqrt{n}\,(\bar{X}_n - \mu)$$

$$V[Z] = n\,V[\bar{X}_n - \mu]$$

$$= n\,\frac{\sigma^2}{n} = \sigma^2$$

$$\sqrt{n}\,(\bar{X}_n - \mu) \xrightarrow{d} N(0, \sigma^2)$$

$$\sqrt{n}\,(\hat{\theta} - \theta) \to N(0, V_\theta)$$

$$V[\hat{\theta}] = V/n$$

$$\frac{\sqrt{n}\,(\hat{\theta} - \theta)}{\sqrt{\hat{V}_\theta}} \xrightarrow{d} N(0,1)$$

H_0: $\underline{\theta = 0}$

$$\frac{\sqrt{n}\,(\hat{\theta} - 0)}{\sqrt{\hat{V}_\theta}} \rightarrow N(0,1)$$