$\mu(x)$ is the best predictor

$Y = \mu(x) + e$ $\quad\quad$ $g(x) \Rightarrow$ any fn of x

$E[(Y - g(x))^2]$ = mean-square error

$$= E[(\mu(x) + e - g(x))^2]$$

$$= E[(e + \mu(x) - g(x))^2]$$

$$= E[e^2] + \overset{=0}{\underline{2E[e(\mu(x) - g(x))]}}$$

$$+ E[(\mu(x) - g(x))^2]$$

$$= E[(Y - \mu(x))^2]$$

$$+ E[(\mu(x) - g(x))^2]$$

$$E[(\mu(x) - g(x))^2] \geq 0$$

Categorical X

$$X \in \{1,2,3\}$$

$$\mu(x) = \begin{cases} \mu_1 & \text{if } X=1 \\ \mu_2 & \text{if } X=2 \\ \mu_3 & \text{if } X=3 \end{cases}$$

$$X_2 = \begin{cases} 1 & \text{if } X=2 \\ 0 & \text{if } X\neq 2 \end{cases}$$

$$X_3 = \begin{cases} 1 & \text{if } X=3 \\ 0 & \text{if } X\neq 3 \end{cases}$$

$$\mu(x) = \mu_2 X_2 + \mu_3 X_3 + \underline{\mu_1}(1 - \underline{X_2} - X_3)$$

$$= \mu_1 + X_2(\mu_2 - \mu_1) + X_3(\mu_3 - \mu_1)$$

$$\beta_0 + X_2\beta_1 + X_3\beta_2$$

"saturated"