Two binary X_1, X_2

$$\mu(x_1; x_2) = (1-x_1)(1-x_2)\mu_{00}$$
$$+ x_1(1-x_2)\mu_{10} + (1-x_1)x_2\mu_{01}$$
$$+ x_1 x_2 \mu_{11}$$

$$= \mu_{00}(1 - x_2 - x_1 + x_1 x_2)$$
$$+ x_1 \mu_{10} - x_1 x_2 \mu_{10}$$
$$+ x_2 \mu_{01} - x_1 x_2 \mu_{01}$$
$$+ x_1 x_2 \mu_{11}$$

$$= \overset{\beta_0}{\mu_{00}} + x_1 \overset{\beta_1}{(\mu_{10} - \mu_{00})}$$
$$+ x_2 \overset{\beta_2}{(\mu_{01} - \mu_{00})}$$
$$+ x_1 x_2 \overset{\beta_3}{(\mu_{11} - \mu_{01} - \mu_{10} + \mu_{00})}$$

3- binary

$$Y \sim X_1 + X_2 + X_3$$

$$+ X_1{:}X_2 + X_1{:}X_3 + X_2{:}X_3$$

$$+ X_1{:}X_2{:}X_3$$

Saturated

Deriving the BLP

$$\underset{}{\arg\min}\ \underbrace{\overset{S(b_1, b_0)}{=} E\left[(Y - (b_0 + b_1 X))^2\right]}$$

$$S(b_0, b_1) = E[Y^2] - 2E[Y(b_0 + b_1 X)]$$
$$+ E[(b_0 + b_1 X)^2]$$

$$= E[Y^2] - 2b_0 \mu_y - 2b_1 E[XY]$$
$$+ b_0^2 + 2b_1 b_0 \mu_x + \underline{b_1^2 E[X^2]}$$

$$0 = \frac{\partial S}{\partial b_0} = -2\mu_y + 2b_0 + \underline{2b_1\mu_x}$$

$$b_0 = \mu_y \underline{- \mu_x b_1}$$

$$
\begin{aligned}
S(b_1) = {} & E[Y^2] - 2\mu_y^2 + 2\mu_x\mu_y b_1 \\
& - 2b_1 E[XY] + \mu_y^2 - \cancel{2\mu_y\mu_x b_1} \\
& + \mu_x^2 b_1^2 + \cancel{2b_1\mu_x\mu_y} - 2b_1^2\mu_x^2 \\
& + b_1^2 E[X^2]
\end{aligned}
$$

$$
\begin{aligned}
= {} & E[Y^2] - \mu_y^2 - 2b_1\left(E[XY] - \mu_x\mu_y\right) \\
& + b_1^2\left(E[X^2] - \mu_x^2\right)
\end{aligned}
$$

$$
\begin{aligned}
= {} & V[Y] - 2b_1\left(\mathrm{Cov}(X,Y)\right) \\
& + b_1^2 V[X]
\end{aligned}
$$

$$
0 = \frac{\partial S}{\partial b_1} = -2\,\mathrm{Cov}(X,Y) + 2b_1 V[X]
$$

$$b_1 = \frac{Cov(\cancel{x}, Y)}{V[x]}$$

$$\mathbf{X} = (1, x)'$$

$$\mathbf{X}\mathbf{X}' = \begin{pmatrix} 1 \\ x \end{pmatrix} \begin{pmatrix} 1 & x \end{pmatrix}$$

$$= \begin{pmatrix} 1 & x \\ x & x^2 \end{pmatrix}$$

$$E[\mathbf{X}\mathbf{X}'] = \begin{pmatrix} 1 & \mu_x \\ \mu_x & E[x^2] \end{pmatrix}$$

$$E[X^T Y] = \begin{pmatrix} E[Y] \\ E[X Y] \end{pmatrix}$$