

16. Clustered and Panel Data

Spring 2021

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Gov 2002 (Harvard)

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- **Panel** and **clustered** data are two common non-iid data.
- Panel data also holds hope for removing unmeasured heterogeneity.

1/ Panel Data

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 - provide benefits more efficiently
 - possess some cultural trait correlated with better health outcomes
- If we have data on countries over time, can we make any progress in spite of these problems?

Ross data

```
ross <- foreign::read.dta("../assets/ross-democracy.dta")
head(ross[,c("cty_name", "year", "democracy", "infmort_unicef")])
```

```
##      cty_name year democracy infmort_unicef
## 1 Afghanistan 1965          0           230
## 2 Afghanistan 1966          0            NA
## 3 Afghanistan 1967          0            NA
## 4 Afghanistan 1968          0            NA
## 5 Afghanistan 1969          0            NA
## 6 Afghanistan 1970          0           215
```

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- **Time series, cross-sectional (TSCS) data:** smaller n , large T (a political science term, mostly)

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- Assume that if we could measure c_i , we would have the correct CEF:

$$\mathbb{E}[u_{it} \mid \mathbf{X}_{it}, c_i] = 0 \quad \implies \quad \mathbb{E}[Y_{it} \mid \mathbf{X}_{it}, c_i] = \mathbf{X}'_{it}\boldsymbol{\beta} + c_i$$

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 2. Errors might be correlated with the covariates
- Both problems arise out of ignoring the **unmeasured heterogeneity** inherent in c_i

Pooled OLS with Ross data

```
pooled.mod <- lm(log(kidmort_unicef) ~ democracy + log(GDPcur),  
                 data = ross)  
summary(pooled.mod)
```

```
##  
## Coefficients:  
##           Estimate Std. Error t value Pr(>|t|)  
## (Intercept)  9.7640      0.3449   28.3 <2e-16 ***  
## democracy   -0.9552      0.0698  -13.7 <2e-16 ***  
## log(GDPcur) -0.2283      0.0155  -14.8 <2e-16 ***  
## ---  
## Signif. codes:  
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1  
##  
## Residual standard error: 0.795 on 646 degrees of freedom  
## (5773 observations deleted due to missingness)  
## Multiple R-squared:  0.504, Adjusted R-squared:  0.503  
## F-statistic: 329 on 2 and 646 DF, p-value: <2e-16
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- Pooled OLS will be inconsistent for the CEF parameters, β .

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- Two approaches that leverage repeated observations:
 - **Differencing** look at changes over time.
 - **Fixed effects** look at relationships within units.

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- Under these assumptions, pooled OLS on the differences is consistent.

3/ Fixed Effects Methods

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- This regression is sometimes called the “between regression”

Within transformation

- **Fixed effect** or **within transformation**:

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- c_i drops out because its within-unit mean is itself (time-constant).
- If we write $\ddot{Y}_{it} = Y_{it} - \bar{Y}_i$, then we can write this more compactly as:

$$\ddot{Y}_{it} = \ddot{\mathbf{X}}'_{it}\boldsymbol{\beta} + \ddot{u}_{it}$$

Fixed effects with Ross data

```
library(lfe)
fe.mod <- lfe::felm(log(kidmort_unicef) ~ democracy + log(GDPcur) | id, data = ross)
summary(fe.mod)

##
## Call:
## lfe::felm(formula = log(kidmort_unicef) ~ democracy + log(GDPcur) | id, data = ross)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.7049 -0.1166  0.0063  0.1222  0.7575
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## democracy      -0.1432     0.0335  -4.28 0.000023 ***
## log(GDPcur)    -0.3752     0.0113 -33.12 < 2e-16 ***
## ---
## Signif. codes:
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.219 on 481 degrees of freedom
## (5773 observations deleted due to missingness)
## Multiple R-squared(full model): 0.972 Adjusted R-squared: 0.962
## Multiple R-squared(proj model): 0.718 Adjusted R-squared: 0.621
## F-statistic(full model): 100 on 167 and 481 DF, p-value: <2e-16
## F-statistic(proj model): 613 on 2 and 481 DF, p-value: <2e-16
```

Strict exogeneity

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- Rules out lagged dependent variables, since $Y_{i,t-1}$ is a function of $u_{i,t-1}$.

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 - Any time-constant variable gets “absorbed” by the fixed effect.
- Can include interactions between time-constant and time-varying variables, but lower order term of the time-constant variables get absorbed by fixed effects too.

Time-constant variables

- Pooled model with a time-constant variable, proportion Islamic:

```
library(lmtest)
p.mod <- felm(log(kidmort_unicef) ~ democracy + log(GDPcur) + islam, data = ross)
coeftest(p.mod)
```

```
##
## t test of coefficients:
##
##           Estimate Std. Error t value Pr(>|t|)
## (Intercept) 10.30608    0.35952   28.67 < 2e-16 ***
## democracy   -0.80234    0.07767  -10.33 < 2e-16 ***
## log(GDPcur) -0.25497    0.01607  -15.87 < 2e-16 ***
## islam        0.00343    0.00091    3.77 0.00018 ***
## ---
## Signif. codes:
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Time-constant variables

- FE model, where the islam variable drops out, along with the intercept:

```
fe.mod2 <- fe1m(log(kidmort_unicef) ~ democracy + log(GDPcur) + islam | id, data = ro
coeftest(fe.mod2)
```

```
##
## t test of coefficients:
##
##           Estimate Std. Error t value Pr(>|t|)
## democracy   -0.1297    0.0359   -3.62  0.00033 ***
## log(GDPcur)  -0.3800    0.0118  -32.07 < 2e-16 ***
## islam                NA         NA      NA      NA
## ---
## Signif. codes:
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

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 - Usually better to use dedicated software like **lfe** package in R.

Example with Ross data

```
library(lmtest)
lsdv.mod <- lm(log(kidmort_unicef) ~ democracy + log(GDPcur) + as.factor(id),
               data = ross)
coeftest(lsdv.mod)[1:6,]
```

##	Estimate	Std. Error	t value	Pr(> t)
## (Intercept)	13.764	0.2660	51.75	1.01e-198
## democracy	-0.143	0.0335	-4.28	2.30e-05
## log(GDPcur)	-0.375	0.0113	-33.12	3.49e-126
## as.factor(id)AGO	0.300	0.1677	1.79	7.45e-02
## as.factor(id)ALB	-1.931	0.1901	-10.16	4.39e-22
## as.factor(id)ARE	-1.876	0.1702	-11.02	2.39e-25

```
coeftest(fe.mod)[1:2,]
```

##	Estimate	Std. Error	t value	Pr(> t)
## democracy	-0.143	0.0335	-4.28	2.30e-05
## log(GDPcur)	-0.375	0.0113	-33.12	3.49e-126

4/ Clustering

Clustered dependence: intuition

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 - SEs are going to be wrong.
- Called **clustering** or **clustered dependence**

Clustered dependence: notation

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- Units are (usually) belong to a single cluster:
 - voters in households
 - individuals in states
 - students in classes
 - rulings in judges
- Outcome varies at the unit-level, Y_{ig} and the main independent variable varies at the cluster level, X_g .

Clustered dependence: example model

$$\begin{aligned} Y_{ig} &= \beta_0 + X_g \beta_1 + v_{ig} \\ &= \beta_0 + X_g \beta_1 + c_g + u_{ig} \end{aligned}$$

- u_{ig} unit error component with $\mathbb{V}[u_{ig}|X_g] = \sigma_u^2$

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 - $\rightsquigarrow \mathbb{V}[v_{ig}|X_g] = \sigma_c^2 + \sigma_u^2$
- What if we ignore this structure and just use v_{ig} as the error?

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$$\text{Cov}[v_{ig}, v_{sg}] = \sigma_c^2$$

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- Zero covariance of two units i and s in different clusters g and k :

$$\text{Cov}[v_{ig}, v_{sk}] = 0$$

Example covariance matrix

$$\bullet \mathbf{v}' = \begin{bmatrix} v_{1,1} & v_{2,1} & v_{3,1} & v_{4,2} & v_{5,2} & v_{6,2} \end{bmatrix}$$

Example covariance matrix

- $\mathbf{v}' = [v_{1,1} \quad v_{2,1} \quad v_{3,1} \quad v_{4,2} \quad v_{5,2} \quad v_{6,2}]$
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$$\mathbb{V}[\mathbf{v}|\mathbf{X}] = \begin{bmatrix} \sigma_c^2 + \sigma_u^2 & \sigma_c^2 & \sigma_c^2 & 0 & 0 & 0 \\ \sigma_c^2 & \sigma_c^2 + \sigma_u^2 & \sigma_c^2 & 0 & 0 & 0 \\ \sigma_c^2 & \sigma_c^2 & \sigma_c^2 + \sigma_u^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_c^2 + \sigma_u^2 & \sigma_c^2 & \sigma_c^2 \\ 0 & 0 & 0 & \sigma_c^2 & \sigma_c^2 + \sigma_u^2 & \sigma_c^2 \\ 0 & 0 & 0 & \sigma_c^2 & \sigma_c^2 & \sigma_c^2 + \sigma_u^2 \end{bmatrix}$$

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$$\mathbb{V}[\mathbf{v}|\mathbf{X}] = \begin{bmatrix} \sigma_c^2 + \sigma_u^2 & \sigma_c^2 & \sigma_c^2 & 0 & 0 & 0 \\ \sigma_c^2 & \sigma_c^2 + \sigma_u^2 & \sigma_c^2 & 0 & 0 & 0 \\ \sigma_c^2 & \sigma_c^2 & \sigma_c^2 + \sigma_u^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_c^2 + \sigma_u^2 & \sigma_c^2 & \sigma_c^2 \\ 0 & 0 & 0 & \sigma_c^2 & \sigma_c^2 + \sigma_u^2 & \sigma_c^2 \\ 0 & 0 & 0 & \sigma_c^2 & \sigma_c^2 & \sigma_c^2 + \sigma_u^2 \end{bmatrix}$$

- Variance matrix under i.i.d.:

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$$Y_{ig} = \beta_0 + X_g \beta_1 + c_g + u_{ig}$$

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- True variance will be higher than conventional when within-cluster correlation is positive, $\rho_c > 0$.

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- We can write the OLS estimator as:

$$\hat{\boldsymbol{\beta}} = \left(\sum_{g=1}^m \mathbb{X}'_g \mathbb{X}_g \right) \left(\sum_{g=1}^m \mathbb{X}'_g \mathbf{Y}_g \right)$$

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- With small-sample adjustment (reported by most software):

$$\hat{\mathbf{V}}_{CL1}[\hat{\boldsymbol{\beta}}] = \frac{m}{m-1} \frac{n-1}{n-k} (\mathbb{X}'\mathbb{X})^{-1} \left(\sum_{g=1}^m \mathbb{X}'_g \hat{\mathbf{v}}_g \hat{\mathbf{v}}'_g \mathbb{X}_g \right) (\mathbb{X}'\mathbb{X})^{-1}$$

Example: Gerber, Green, Larimer

Dear Registered Voter:

WHAT IF YOUR NEIGHBORS KNEW WHETHER YOU VOTED?

Why do so many people fail to vote? We've been talking about the problem for years, but it only seems to get worse. This year, we're taking a new approach. We're sending this mailing to you and your neighbors to publicize who does and does not vote.

The chart shows the names of some of your neighbors, showing which have voted in the past. After the August 8 election, we intend to mail an updated chart. You and your neighbors will all know who voted and who did not.

DO YOUR CIVIC DUTY — VOTE!

MAPLE DR	Aug 04	Nov 04	Aug 06
9995 JOSEPH JAMES SMITH	Voted	Voted	_____
9995 JENNIFER KAY SMITH		Voted	_____
9997 RICHARD B JACKSON		Voted	_____
9999 KATHY MARIE JACKSON		Voted	_____

Social pressure model

```
load("../assets/gerber_green_larimer.RData")
library(lmtest)
social$voted <- 1 * (social$voted == "Yes")
social$treatment <- factor(
  social$treatment,
  levels = c("Control", "Hawthorne", "Civic Duty", "Neighbors", "Self")
)
mod1 <- lm(voted ~ treatment, data = social)
coeftest(mod1)
```

```
##
## t test of coefficients:
##
##              Estimate Std. Error t value
## (Intercept)    0.29664    0.00106  279.53
## treatmentHawthorne  0.02574    0.00260    9.90
## treatmentCivic Duty 0.01790    0.00260    6.88
## treatmentNeighbors  0.08131    0.00260   31.26
## treatmentSelf      0.04851    0.00260   18.66
##              Pr(>|t|)
## (Intercept)      < 2e-16 ***
## treatmentHawthorne < 2e-16 ***
## treatmentCivic Duty 5.8e-12 ***
## treatmentNeighbors < 2e-16 ***
## treatmentSelf     < 2e-16 ***
## ---
```

Social pressure model, CRSEs

```
library(sandwich)
coeftest(mod1, vcov = sandwich::vcovCL(mod1, cluster = social$hh_id))
```

```
##
## t test of coefficients:
##
##           Estimate Std. Error t value
## (Intercept)      0.29664    0.00131  226.52
## treatmentHawthorne 0.02574    0.00326    7.90
## treatmentCivic Duty 0.01790    0.00324    5.53
## treatmentNeighbors 0.08131    0.00337   24.13
## treatmentSelf      0.04851    0.00330   14.70
##
##           Pr(>|t|)
## (Intercept)      < 2e-16 ***
## treatmentHawthorne 2.8e-15 ***
## treatmentCivic Duty 3.2e-08 ***
## treatmentNeighbors < 2e-16 ***
## treatmentSelf      < 2e-16 ***
## ---
## Signif. codes:
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

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- Consistency of the CRSE are in the number of groups, not the number of individuals
 - CRSEs can be incorrect with a small (< 50 maybe) number of clusters